Optimum wave regime for lift-based wave energy converters

M. Folley, and P. Lamont-Kane

Abstract—**Lift-based Wave Energy Converters (WECs) have a number of attractive features, including the potential for unidirectional rotation, simplifying power take-off and reduction in wave loads by reducing generation of circulation, increasing survivability. The common assumption of small body, small amplitude response, together with the Haskinds Relationship is used to determine the optimum motion for a lift-based WEC to maximise power capture. It is shown that whilst for a 2D hydrofoil in deep water the optimum motion is circular, the optimum motion for a finite-width hydrofoil is generally elliptical due to differences in the hydrodynamic damping coefficients associated with the vertical and horizontal motions of the hydrofoil. It is shown that more circular hydrofoil motion can be achieved by utilising the elliptical motion of the water particles in shallow water. This occurs because the increased horizontal water particle motion in shallow water results in an increase in the wave-induced lift force associated with horizontal fluid particle motions, and thus a reduction in the optimum amplitude of motion in this direction. Preliminary calculations suggest that for a 30 metre wide hydrofoil in wave periods of about 10 seconds, the ideal water depth (where the optimum hydrofoil motion is circular) occurs at around 25 metres, which is a highly utilisable water depth. Other advantages of deployment in shallower water include an improvement in the alignment of the waves parallel to the hydrofoil and a reduction in the structural task associated with reacting against the seabed.**

Keywords— **hydrodynamics, lift force, water depth, wave energy**

I. NOMENCLATURE

Symbol	Quantity	Unit
L	Lift force per unit span	N/m
Γ	Strength of circulation around hydrofoil	m^2/s
V_i	Relative incident velocity on hydrofoil	m/s
B	Radiation damping coefficient	Ns/m
ω	Wave frequency	rad/s
k	Wave number	1/m
ρ	Water density	kg/m^3
g	Gravitational acceleration	m/s^2
A	Wave amplitude	m
h	Water depth	m
R	Radius of hydrofoil rotation	m
m	Radiation pattern ratio	-
p	Water particle velocity ratio	-
z	Distance below surface	m
β	Direction of wave propagation	rad
F	Wave excitation force	N
P_i	Incident wave power density	W/m
P	Power capture	W
$D(kh)$	Depth function	-
λ	Wavelength	m
V	Hydrofoil velocity	m/s
v	Wave-induced water particle velocity	m/s
Subscripts		
x	Force / motion in horizontal direction	
z	Force / motion in vertical direction	

II. INTRODUCTION

Wave Energy Converters (WECs) extract energy from the waves by coupling to the wave-induced water particle motions. In the vast majority of wave energy converters this coupling is through buoyancy (Froude-Krylov) and/or diffraction wave forces, so that almost all the research in the hydrodynamics of wave energy converters has focused on the characteristics of these forces. Thus, there is a vast amount of knowledge about how these wave forces vary may vary with body characteristics, e.g., size, shape, etc. and the wave characteristics, e.g., frequency, spectral shape, water depth, etc. This knowledge can then be used to support the optimal design of wave energy converters that utilise buoyancy and/or diffraction forces.

In comparison, only a small number of wave energy converters have been proposed that couple with wave-induced lift forces [1], so that there has been very little research into the characterisation of these forces. Thus, there is a general lack of knowledge about how wave-induced lift forces are affected by the body and wave characteristics, which means there is limited support for the design of lift-based wave energy converters.

The reasons for the previously limited interest in lift-based wave energy converters are unclear, although

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research has shown and continues to show that the utilisation of wave-induced lift forces offers many potentially advantageous characteristics. However, to determine the potential for lift-based wave energy converters it is important to develop a significant understanding of the fundamental hydrodynamics that govern their coupling with the incident water waves.

This paper investigates one aspect of these fundamental hydrodynamics, which is the effect of wave regime, and thus water depth, on the design of a lift-based wave energy converter; specifically, the optimum motion of the hydrofoil required to maximise power capture. Although not a requirement, it may be expected that a circular path for the hydrofoil will be advantageous for designs that involve rotation of the hydrofoil about a central axis. A circular path means that the hydrofoil can be mounted in a simple fixed radius arm as for the CycWEC [2]. A sketch of this type of lift-based wave energy converter is provided in Figure 1.

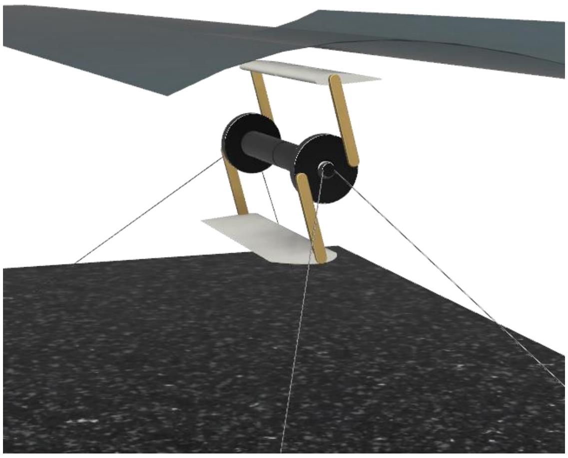

Figure 1: An example of the type of lift-based wave energy converter considered in this paper (Source: [1])

The optimum hydrofoil motion is identified by first considering the hydrodynamics of lift-based wave energy converters (Section III) and its relationship to the far-field hydrodynamics (Section IV). These are then used to determine the optimum motion of the hydrofoil (Section V) and how this is affected by the water depth and thus wave regime (Section VI). Finally, conclusions on the implications of this investigation are drawn (Section VII) and further work identified (Section VIII).

III. HYDRODYNAMICS OF LIFT-BASED WAVE ENERGY CONVERTERS

In common with other wave energy converters, a full representation of the hydrodynamics of a lift-based wave energy converter is complex and challenging. However, experience has shown that a good understanding of the general hydrodynamics of wave energy converters can be achieved by making a number of assumptions which simplify the problem without significantly affecting device response characteristics. For example, the approximation of ocean waves as the sum of single frequency sinusoidal waves is a simplifying assumption as ocean waves are clearly much more complex; however, this assumption underlies much of the current understanding of the design of wave energy converters where they have been developed using frequency-domain analyses [3]. Moreover, harmonic analysis, based on this assumption together with a linearisation of the hydrodynamic forces, has been found to support the successful design of many wave energy converter concepts. Thus, it is important to recognise that although these wave energy converter representations contain gross assumptions, they are often useful in supporting design. However, it is also important to recognise that they are typically less useful in actually calculating the potential performance of the wave energy converter. Consequently, they are generally best considered as indicators of direction for design choices and should not be used as tools for numerical optimisation (except in cases where a suitable alternative higher fidelity model does not exist).

The gross assumptions and justifications in this analysis of lift-based wave energy converters are:

- The waves can be represented as single frequency Airy waves.
    - This is at the core of almost all fundamental wave energy converter theories, and in general has been found to enable a good understanding of the relevant hydrodynamics to be developed (see for example [4]–[6]). It is expected that this will also be the case for lift-based wave energy converters.
- The hydrofoil(s) moves along a circular path in a plane parallel to the direction of wave propagation and around a fixed point.
    - This is the configuration for CycWEC and likely to be typical for lift-based WECs.
- The wave-induced velocity acting on the hydrofoil is equal to the water particle velocity at the centre of hydrofoil rotation.
    - The water particle velocity at the centre of rotation typically differs from the average water particle along the path of the hydrofoil by only a few percent.
- The lift forces are proportional to the angle of attack on the hydrofoil and the instantaneous relative velocity squared.
    - This is a good approximation prior to stall, which if the device is operating correctly should typically be satisfied.
- The phase between the wave-induced velocity and hydrofoil velocity is fixed (phase-locking).
    - This is the control strategy for CycWEC and is equivalent to maintaining the correct phase between the body motion and wave force.
- The pitch of the hydrofoil is fixed.
    - Although it may be possible to vary the hydrofoil pitch during operation the solution with a fixed

pitch is considered to give a good indication of the expected response to support design.

A fundamental characteristic for lift-based wave energy converters is the generation of the lift force. The Kutta-Joukowski theorem relates the lift generated by a body to the circulation around the body [7], which is as given by

$$L = \rho V_i \Gamma \tag{1}$$

Specifically, this shows that the lift force is proportional to the product of the circulation and the incident relative velocity. In addition, a correctly operating hydrofoil satisfies the Kutta condition, which requires that the velocity of the fluids along the lower and upper surfaces of the hydrofoil must meet smoothly. For small, pre-stall, angles of attack this requires that the circulation be proportional to the product of the angle of attack and the incident velocity. Thus, the theoretical lift force is proportional to the product of the angle of attack and incident velocity squared. In reality, the actual lift force deviates from this theoretical prediction due to non-idealised flow, but in general it is found that this is a reasonably good approximation for hydrofoils that have not stalled [7].

The use of the wave-induced velocity at the hydrofoil at the centre of rotation rather than the velocity at the location of the hydrofoil is an assumption that is reasonable provided that the radius of the hydrofoil motion is small relative to the wavelength. Small body motions are a standard assumption for the application of linear potential theory and are widely used with success in the generation of hydrodynamic coefficients for many wave energy converters. Although the motions of the hydrofoil may be larger than the motions for many other wave energy converters it is expected to be a similar first-order approximation which allows the fundamental interaction of the hydrofoil with the waves to be understood. Further investigation of the suitability of this assumption for typical lift-based wave energy converters is planned for completion as further work at a later stage of development.

Although phase-locking of the hydrofoil rotation, a circular hydrofoil path and a fixed hydrofoil pitch are not fundamental requirements for the operation of a lift-based wave energy converter, they are expected to be reasonably representative characteristics of many possible concepts. Thus, it is anticipated that this analysis can help to provide a baseline against which other concepts may be referenced. This is analogous to the assumption of a constant linear damping coefficient in models of more traditional wave energy converters; they may not represent the actual wave energy converter damping, but the analysis can help to provide understanding of how the wave energy converter needs to be designed.

Some initial 2D models of lift-based wave energy converters are based on a potential-flow solution, where the circulation generated by the hydrofoil has been imposed [8], [9]. These have shown that it is possible to absorb almost 100% of the incident wave energy. However, the absence of drag losses in the model means that the model cannot help in identifying how best to generate the circulation. Alternatively, using a standard hydrofoil model, where the lift and drag forces depend on the angle of attack and relative velocity, together with far-field relationships, it is possible to show that for any incident wave there is an optimum angle of attack and relative velocity that maximises the power capture [10]. That is, where the angle of attack and/or relative velocity differ from their optimum values there is a reduction in power capture.

IV. FAR-FIELD HYDRODYNAMICS

Far-field hydrodynamics can be used to calculate the energy that is absorbed from the waves by considering the energy flux in/out of a control volume whose boundary is a large distance from the wave energy converter [11], [12]. The wave energy entering the control volume depends on the incident wave field, whilst the amount of energy leaving the control volume is reduced by the destructive interference of the waves diffracted/radiated by the wave energy converter with the incident wave field. Importantly, the maximum absorbed power solely depends on the azimuthal variation in the magnitude and phase of the far-field diffracted/radiated waves from the wave energy converter, not how they are generated.

A particularly important far-field characteristic relevant to wave energy converters is the Haskind Relationship [3], [13], which relates the radiation (hydrodynamic) damping with the complex amplitude of the excitation force.

$$B = \frac{\omega k}{4\pi\rho g^2 D(kh)|A|^2} \int_{-\pi}^{\pi} |F(\beta)|^2 \, d\beta \tag{2}$$

The Haskind Relationship essentially means that a far-field wave will only be radiated at a particular angle if incident waves from that angle result in an excitation force. In addition, it is also possible to show that the maximum power capture of a single mode of a wave energy converter for waves coming from the direction β is given by.

$$P(\beta) = \frac{|F(\beta)|^2}{8B} \tag{3}$$

These two relationships are combined to produce the well-known 'point absorber' theory result that for a body that is small relative to the incident wavelength [14].

$$P = \frac{n\lambda}{2\pi} P_i \tag{4}$$

where $n = 1$ for the heave mode, $n = 2$ for the surge mode (aligned with the direction of wave propagation), and $n = 3$ for the combination of heave and surge modes. This difference occurs because for a small body the surge wave force is proportional to the cosine of the direction β. That is:

$$B_x = \frac{\omega k |F_x|^2}{4\pi\rho g^2 D(kh)|A|^2}\int_{-\pi}^{\pi}\cos^2\beta\, d\beta \quad (5)$$

$$B_x = \frac{1}{2}\frac{\omega k |F_x|^2}{4\pi\rho g^2 D(kh)|A|^2} \quad (6)$$

The radiation patterns for these two modes are shown in Figure 2.

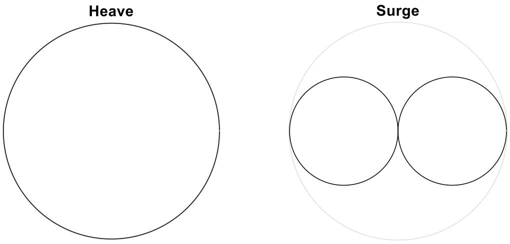

Figure 2: Radiation patterns for heave and surge modes

Whilst for heave, because the force is independent of direction, the radiation damping coefficient is given by:

$$B_z = \frac{\omega k |F_z|^2}{4\pi\rho g^2 D(kh)|A|^2} \quad (7)$$

Thus, the radiation damping coefficient for a small body in surge has half the value of that in heave for the same unit incident wave force. A more general case would be that the ratio of the radiation damping coefficients is given by:

$$\frac{B_z}{B_x} = m\frac{|F_z|^2}{|F_x|^2} \quad (8)$$

where m is a factor that accounts for the difference in the radiation patterns for the two modes, noting that $m = 2$ for a small body. Note that this far-field analysis is relevant to all wave forces, including lift. That is, in the far-field there is no distinction between the hydrodynamics of a heaving buoy, surging flap or rotating hydrofoil, except to the extent that they may generate different radiation patterns.

Now, consider a hydrofoil that is moving horizontally and interacting with the vertical wave-induced water particle motions to generate a lift force. For a hydrofoil of short span (relative to the wavelength) the lift force generated will be independent of the propagation direction of the incident waves. However, a short-span hydrofoil moving vertically will only interact with the horizontal wave-induced water particle motion that is orthogonal to the velocity of the hydrofoil, so that there is a cosine variation in the wave-induced lift force with the direction of wave propagation. Applying these two relationships to (2) indicates that the radiation damping coefficient for a short-span hydrofoil moving horizontally is double that of a similar hydrofoil moving vertically, equivalent to the damping coefficients calculated for the heave and surge modes in (6) and (7).

As the span of the hydrofoil increases (relative to the wavelength) the azimuthal variation of the wave-induced lift forces, and thus the radiation damping coefficients, will change for both the horizontal and vertical wave-induced water particle motions. In the limit as the span of the hydrofoil tends to infinity (the 2D case), the azimuthal variation of the wave-induced lift force will tend to a Dirac delta function orthogonal to the span of the hydrofoil. Moreover, the same Dirac Delta function will apply to both horizontal and vertical motions so that a 2D hydrofoil will have the same radiation damping coefficient for both horizontal and vertical wave-induced water particle motions.

V. OPTIMUM HYDROFOIL MOTION

The far-field hydrodynamics described in Section IV can also be used to define the optimum hydrofoil motion [14]. That is, to achieve the maximum power capture given in (3) then the hydrofoil velocity must be:

$$|V| = \frac{|F|}{2B} \quad (9)$$

For a hydrofoil that is rotating about a point with the wave frequency the velocity is equal to the product of the wave frequency and the radius. Thus, the ratio of the optimum radii of hydrofoil motion, see Figure 3, is given by:

$$\frac{R_x}{R_z} = \frac{|V_x|}{|V_z|} = \frac{|F_x|}{|F_z|}\frac{B_z}{B_x} \quad (10)$$

However, Haskins Relation means that the radiation damping can be defined in terms of the wave force and the radiation patterns, and the ratio of the damping coefficients is given by (8), which when combined with (10) gives:

$$\frac{R_x}{R_z} = m\frac{|F_z|}{|F_x|} \quad (11)$$

In deep water and for a given depth the wave-induced water particle motions will be the same in both the horizontal and vertical directions. To a first order approximation this means that the wave-induced lift forces will also be same in the horizontal and vertical directions ($|F_x| = |F_z|$). Then, for a 2D hydrofoil the radiation patterns are the same for both horizontal and vertical motions ($m = 1$), which means that the optimum hydrofoil velocity will be the same for both directions. Thus, for a 2D hydrofoil excited by deep water regular waves the optimum path of the hydrofoil is circular.

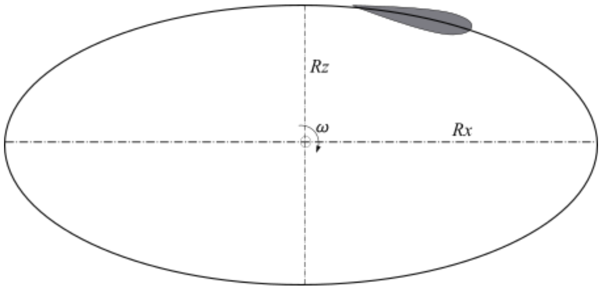

Figure 3: Elliptic hydrofoil motion

Remaining in deep water but now consider a finite-span hydrofoil. In this case, the wave-induced lift forces in the horizontal and vertical directions may be expected to remain approximately equal (for a given hydrofoil pitch angle), however, the damping coefficients will be different due to the different azimuthal variations in their radiated waves ($B_x \neq B_z$). This means that the optimum motion of lift-based wave energy converter with a finite-span hydrofoil will not be circular but elliptical. In the limit as the span of the hydrofoil becomes small relative to the wavelength the ratio of radiation damping coefficients tends to two ($m = 2$) as can be derived from (6) and (7). Then, from (10), the optimum horizontal to vertical hydrofoil motion is equal to two. Typically, a lift-based wave energy converter may be expected to have a hydrofoil span that is neither very small relative to the wavelength so that it can be represented as a point source, nor so large that it can be assumed to be of infinite length (2D). Thus, in general the ratio of optimum horizontal to vertical hydrofoil motion in deep water may be expected to be between one and two, that is, elliptical.

VI. EFFECT OF WATER DEPTH

As water depth decreases the wave-induced water particle velocities change from having a circular path to an elliptical path. That is, the horizontal wave-induced water particle motions/velocities become larger than the vertical motions/velocities. The ratio varies with depth, so that there is more difference between the horizontal and vertical wave-induced velocities at the seabed than there is at the water surface. The general expression for the ratio of the horizontal to vertical water particle velocities (p) can be obtained from Airy wave theory and is given by:

$$p = \frac{v_x}{v_z} = \frac{\cosh\big(k(h-z)\big)}{\sinh\big(k(h-z)\big)} \tag{12}$$

Thus, the ratio of the horizontal to vertical water particle motions depends on the water depth relative to the wavelength and is shown graphically in Figure 4 as a variation with water depth and wave period at mid-depth ($z = h/2$). For example, at a wave period of 11 seconds in a water depth of 25 metres the mid-depth horizontal wave-induced water particle velocity is approximately double that of the vertical wave-induced velocity.

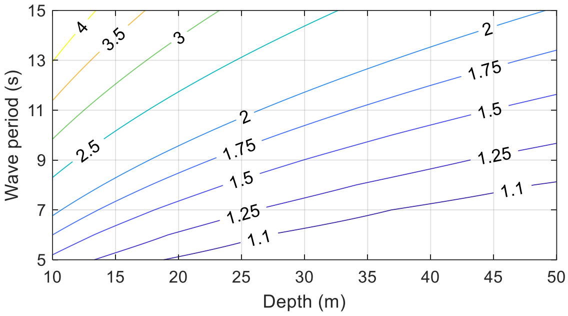

Figure 4: Ratio of horizontal to vertical wave-induced water particle velocities at mid-depth for different wave periods and water depths

The difference between the horizontal and vertical wave induced water particle motions mean that unlike deep water, the lift force will vary through the wave cycle due to changes in the incident wave-induced water particle velocities. The difference can be approximated by defining the lift force as proportional to the product of the angle of attack and the relative velocity squared (justification provided in Section II). Provided that the wave amplitude is small relative to the radius of rotation of the hydrofoil (Airy wave theory), then relative velocity will remain constant during the whole wave cycle (as it is dominated by the hydrofoil motion) and the angle of attack will vary in proportion to the ratio of horizontal and vertical wave-induced water particle velocities. Thus, the ratio of the horizontal and vertical lift forces is equal to the ratio of the horizontal and vertical wave-induced water particle motions. That is:

$$\frac{|F_x|}{|F_z|} = \frac{v_x}{v_z} = \frac{1}{p} \tag{13}$$

This ratio of lift forces can then be used to determine the ratio of optimum radii of horizontal to vertical hydrofoil motion given in (11), so that:

$$\frac{R_x}{R_z} = \frac{m}{p} \tag{14}$$

This indicates that a circular path for the hydrofoil ($R_x/R_z = 1$) can be achieved if the radiation pattern ratio (m) is the same as the water particle velocity ratio (p).

The effect of this is investigated by first considering the case of a short-span hydrofoil. It has already been shown that this will have a radiation pattern ratio of two ($m = 2$). Thus, the hydrofoil will have a circular path if the water particle velocity ratio is also equal to two ($p = 2$). Figure 4 shows that this can be achieved with a number of different combinations of wave period and water depth.

In general, the hydrofoil will have a finite span so that the radiation pattern ratio will have a value between one and two. Importantly a similar water particle velocity ratio can be achieved with a range of combinations of wave period and water depth are in the region that are likely to be of interest for the exploitation of wave energy ($6 < T < 13$ s and $h > 20$ m). Thus, the ellipticity of the optimum hydrofoil motion that occurs for a finite-span hydrofoil in deep water can be compensated for by selecting an appropriate water depth for a typical "design" wave period which benefits from the effect of elliptical wave motions through improved performance of a device designed to operate with a circular hydrofoil path. Identification of the hydrofoil design that provides the required balance between the effect of elliptical wave-induced water particle motions and the hydrofoil span requires consideration of the lift distribution along the hydrofoil span. However, preliminary calculations suggest that in a North Atlantic wave climate (mean wave period of 10 seconds) a balance can occur with a hydrofoil span of approximately 75 metres deployed in a water depth of approximately 30 metres.

VII. CONCLUSIONS

The results of this study indicate that for the case of a lift-based wave energy converter comprised of a wave-driven rotating hydrofoil of finite-span the optimum path of motion is typically not circular, but rather elliptical. The ellipticity of this optimum path is governed by the span of the hydrofoil relative to the wavelength and the water depth within which it is deployed. However, the results also indicate that the optimum path of the hydrofoil can be circular, which can be achieved by deploying it in a water depth that compensates for the difference in the radiation patterns associated with coupling with the horizontal and vertical water particle motions. The exact water depth in which this compensation occurs depends on the wave period, depth of deployment and hydrofoil span.

Of course, ocean waves do not have a single period and so the optimum hydrofoil path for a real-world device of fixed design characteristics will not always be circular. However, by selecting the water depth so that the optimum path is circular at a typical wave period (sometimes called design wave period) for the wave climate it is possible to minimise the divergence of the optimum path from circular and maximise the potential power capture. This is analogous to the common approach of designing a wave energy converter so that its natural period is equal to the typical (design) wave period; the periods will not always match, but the effect of differences in the periods is expected to be minimised.

Although an increase in the power capture of a lift-based wave energy converter when deployed in shallower water (compared to deep water) is clearly beneficial, it is important to consider other aspects of the design that may also be affected by this shallower deployment location. It is important to recognise that in the water depths that are suggested by this analysis (20 – 30 metres) there is no significant reduction in the exploitable wave resource compared to deep water, and that the reduction in water depth will tend to reduce the directional distribution of the incident waves [15]. This is expected to improve performance as there will be less span-wise variation of the phase of the incident waves so that the angle of attack along the hydrofoil will vary less and thus lead to an improvement in the coupling with the incident waves.

A potentially significant advantage of deploying bottom-mounted lift-based wave energy in shallower water is a reduction in the structural task. The structural task is defined as the product of the applied force and the transmission distance of this force. The amount of structure required to transmit the wave forces to the seabed may be expected to increase dramatically with water depth. Not only because the transmission distance is larger, but also because the over-turning moments from the wave forces will be significantly larger. Indeed, the structural task of transmitting the over-turning moment can be expected to increase with the water depth squared as the supporting structure acts as a cantilever on the seabed. Thus, a move from a typical deep water deployment depth of 50 metres to a deployment depth of 25 metres may be expected to reduce the mass of material used in the support structure by a factor of 4.

It is worth reflecting that this analysis assumes that a promising lift-based wave energy converter will approximate to a circular hydrofoil path with a fixed hydrofoil pitch. Although mechanisms for the generation of elliptical, or close to elliptical, motion exist (e.g., the Trammel of Archimedes, and the Elliptical Drive), they are generally more complex and likely to be thus less attractive for a wave energy converter than circular motion, which can be achieved with simple rotation about a fixed point. This suggests that an optimal circular hydrofoil path is a reasonable objective in lift-based wave energy converter design. However, the assumption of fixed hydrofoil pitch requires more consideration.

Although this analysis assumes a fixed pitch, it is reasonable to consider that the hydrofoil pitch may be both variable and controllable. Obviously, if the pitch is controllable then it could theoretically be modified through the wave cycle to maximise the wave energy absorbed from both the horizontal and vertical wave-induced water particle motions. Indeed, it may be expected for the pitch to be varied to counteract the effects of irregular waves. However, although pitch could be controlled to compensate for the different hydrodynamics in the horizontal and vertical directions, there are two drawbacks of this approach. The principal drawback is that in general a hydrofoil has an pitch that maximises the lift to drag ratio and thus power capture [10]. Variations from this pitch will reduce the maximum power capture and so as far as possible should be avoided. An additional drawback is that controlling the pitch to compensate for the different hydrodynamics in the horizontal and vertical directions is the additional work-load on the pitch controller, which may have implications for its service life.

It is also important to reiterate that linear potential flow theory is generally not sufficient for estimation of the real power capture. Many other factors such as turbulence, vortex shedding, generator characteristics, etc. will also influence the power capture and the design to minimise the cost of energy. However, there is currently no reason to expect that the conclusions regarding the effect of wave regime and water depth would become invalid with the consideration of these effects. Thus, whilst they may affect the dimensions of the final design (that minimises the cost of energy) they do not invalidate the conclusions from the analysis in this paper.

Finally, an incidental, but important, conclusion from the analysis is that the optimal hydrodynamic design of a lift-based wave energy converter cannot be fully achieved using a hydrodynamic analysis based on a 2D representation of the hydrofoil. A 2D representation is not capable of appropriately accounting for the waves radiated by a lift-based wave energy converter resulting in a potential misunderstanding of the optimal design and maximum power capture. Specifically, a 2D representation would suggest that the optimum path of the hydrofoil is circular in deep water whereas it has been shown above that the optimum hydrofoil path of a finite-span hydrofoil in deep water is elliptical. A similar shortcoming with 2D analysis was found for oscillating wave surge converters where the influence of device width on the coupling with the waves cannot be interpreted correctly using a 2D representation of the hydrodynamics [4].

VIII. FURTHER WORK

The analysis within this paper demonstrates that there is an advantage in deploying a lift-based wave energy converter in water depths of 20 – 30 metres to compensate for the different hydrodynamics of the hydrofoil in the horizontal and vertical directions. However, the paper does not calculate the increase in power capture that may be expected, which requires estimation of the radiation pattern ratio and the effect of sub-optimal angle of attack.

Unfortunately, there are no standard tools for determining the radiation pattern for a finite-span hydrofoil in waves. Notwithstanding, the hydrofoil radiation pattern can be estimated by assuming that it is the sum of multiple monopoles or dipoles distributed along the hydrofoil span. The use of multiple monopoles/dipoles to represent a body is a common technique used in many potential flow solvers. In this case the monopoles/dipoles are associated with the waves generated by the hydrofoil and based on the far-field arguments provided in Section IV, the radiated waves associated with the vertical wave-induced water particle velocities should be represented using a monopole and those associated with the horizontal wave-induced wave particle velocities should be represented using a dipole. Then, the strength of the monopoles/dipoles are assumed to vary proportionally with the circulation along the length of the hydrofoil, which can be specified based on current knowledge of the distribution of circulation along hydrofoils.

The effect of sub-optimal angle of attack can be estimated by assuming that the lift force is proportional to the angle of attack and considering the energy flows in and out of the hydrofoil. If the angle of attach (lift force) is too large means that too much energy will be radiated by the wave energy converter resulting in a net reduction in power capture. Conversely, if the angle of attack (lift force) is too small then too little energy will be extracted from the incident waves and again there will be a net reduction in power capture. This is similar to, and based on the same considerations as, the analysis for the constrained motion of wave energy converters [16].

Based on these techniques it should be possible to estimate the change in power capture associated with the water depth. However, clearly these results need to be verified and validated. It is proposed that the results will be verified against a CFD numerical model of the hydrofoil and validated against wave-tank experiments, both of which are currently being planned.

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